# CHAPTER 6 ❑ MATLAB EXERCISES

**1.** Find the kernel and range of the linear transformation given by $T(\mathbf{x}) = A\mathbf{x}$ for these matrices $A$.

(a) $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -1 & -1 & 1 \\ -2 & 0 & -2 \end{bmatrix}$ (b) $A = \begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & -1 & -2 & 2 \\ 1 & 2 & 4 & -5 \end{bmatrix}$

(c) $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 9 & 10 & 11 & 12 \\ 5 & 6 & 7 & 8 \\ -13 & -14 & -15 & -16 \end{bmatrix}$ (d) $A = \begin{bmatrix} 1 & -1 & 1 & -1 & 1 \\ -1 & 1 & -1 & 1 & -1 \end{bmatrix}$

**2.** Let $B$ be the upper trianglular matrix generated by the MATLAB command **B = triu(ones(6)).** Let $A = BB^T - B$ and determine the rank and nullity of the linear transformation

$L: R^6 \rightarrow R^6, L(\mathbf{x}) = A\mathbf{x}.$

**3.** Which of these linear transformations defined by $T(\mathbf{x}) = A\mathbf{x}$ are one-to-one? Which are onto?

(a) $A =$ magic(6) (b) $A =$ hilb(6) (c) $A =$ tril(ones (6))

**4.** Let $T: R^n \rightarrow R^m$ be a linear transformation. Let $B = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ and $B1 = \{\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_m\}$ be bases for $R^n$ and $R^m$, respectively. You can use MATLAB to find the matrix of $T$ relative to the bases $B$ and $B1$ as follows.

(a) Form the matrices $B$ and $B1$ whose *columns* are the given basis vectors.
(b) Let $A$ be the $m \times n$ standard matrix of $T$.
(c) Adjoin $B1$ to $AB$ to form the $m \times (m + n)$ matrix $C$: **C = [B1 A*B]**.
(d) Use **rref(C)** to calculate the reduced row-echelon form of $C$. The $m \times n$ matrix composed of the right-hand $n$ columns of $C$ form the matrix of $T$ relative to the bases $B$ and $B1$.

Use this algorithm to find the matrix of the following linear transformations relative to the given bases.

(a) $T: R^2 \rightarrow R^3$, $T(x, y) = (x + y, x, y)$,
$B = \{(1, -1), (0, 1)\}$, $B1 = \{(1, 1, 0), (0, 1, 1), (1, 0, 1)\}$
(b) $T: R^3 \rightarrow R^2$, $T(x, y, z) = (2x - z, y - 2x)$,
$B = \{(2, 0, 1), (0, 2, 1), (1, 2, 1)\}$, $B1 = \{(1, 1), (2, 0)\}$
(c) $T: R^3 \rightarrow R^4$, $T(x, y, z) = (2x, x + y, y + z, x + z)$,
$B = \{(2, 0, 1), (0, 2, 1), (1, 2, 1)\}$,
$B1 = \{(1, 0, 0, 1), (0, 1, 0, 1), (1, 0, 1, 0), (1, 1, 0, 0)\}$

**5.** Use the results of Exercise 4 to find the image of the given vector $\mathbf{v}$ two ways: first by calculating $T(\mathbf{v}) = A\mathbf{v}$, and second by using the matrix of $T$ relative to the bases $B$ and $B1$.

(a) $\mathbf{v} = (5, 4)$
(b) $\mathbf{v} = (0, -5, 7)$
(c) $\mathbf{v} = (1, -5, 2)$

**6.** Let $B = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ and $B1 = \{\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_n\}$ be two ordered bases for $R^n$. Recall from Section 4.7 that you find the transition matrix $P^{-1}$ from $B$ to $B1$ as follows.

(a) Form the matrices $B$ and $B1$ whose columns are the given basis vectors.

(b) Adjoin $B$ to $B1$, forming the $n \times 2n$ matrix $C$, $\mathbf{C} = [\mathbf{B1\ \ B}]$.

(c) Let $D$ be the reduced row-echelon form of $C$, $\mathbf{D} = \mathbf{rref(C)}$.

(d) $P^{-1}$ is the $n \times n$ matrix consisting of the right-hand $n$ columns of $D$.

Use MATLAB to find the matrix $A1$ of the linear transformation $T: R^n \to R^n$ relative to the basis $B1$.

(a) $T: R^2 \to R^2,\ T(x, y) = (2x - y, y - x)$,
$B1 = \{(1, -2), (0, 3)\}$

(b) $T: R^3 \to R^3,\ T(x, y, z) = (x, x + 2y, x + y + 3z)$,
$B1 = \{(1\ -1, 0), (0, 0, 1), (0, 1, -1)\}$

**7.** Let $B = \{1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ and $B1 = \{(1, 1, 0), (1, -1, 0), (0, 0, 1)\}$ be bases for $R^3$, and let

$$A = \begin{bmatrix} 1 & 3 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

be the matrix of $T: R^3 \to R^3$ relative to $B$, the standard basis.

(a) Find the transition matrix $P$ from $B1$ to $B$.

(b) Find the transition matrix $P^{-1}$ from $B$ to $B1$.

(c) Find $A1$, the matrix of $T$ relative to $B1$.

(d) Let

$$[\mathbf{v}]_{B1} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

and find $[\mathbf{v}]_B$ and $[T(\mathbf{v})]_B$.

(e) Find $[T(\mathbf{v})]_{B1}$ two ways: first as $P^{-1}[T(\mathbf{v})]_B$ and then as $A1[\mathbf{v}]_{B1}$.